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Small sample properties of the regression test of the expectations model of the term structure[1]

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Abstract

The econometric properties of the forecasting equation relating the change of the long term interest rate to the lagged value of the spread are investigated. Due to the extremely low population R^2 of this model it can not be expected that we can produce any convincing empirical evidence against the expectations hypothesis. The results are illustrated with a Monte Carlo experiment. © 1997 Elsevier Science S.A.

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1. Introduction

This paper examines the statistical properties of the regression test of the expectations model of the term structure of interest rates. In this regression the spread between long and short term interest rates is used as a predictor of changes in the long term interest rate. The equation is a direct implication of the linearized version of the expectations model (see Shiller (1979)), which states that

$$R_t = (1-\delta)\sum_{i=0}^{\infty} \delta^i E_t[r_{t+i}] + c, \tag{1}$$

where R_t is the yield to maturity on a default free bond with infinite maturity (the long term interest rate), r_t is the one period short term interest rate, $\delta = (1/1+\rho)$ is a discount factor ($0<\delta<1$), c is a constant risk premium, and E_t denotes conditional expectation with respect to the information at time t. From Eq. (1) it follows that

$$E_t[\Delta R_{t+1}] = \rho(S_t - c), \tag{2}$$

where $S_t = (R_t - r_t)$ is the spread between the long and short term interest rate. If expectations are rational, Eq. (2) has implications for the parameters in the regression model

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[1]This paper contains the results in Appendix 1C of my dissertation Schotman (1989).

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$$\Delta R_{t+1} = \alpha + \beta S_t + \nu_{t+1}, \tag{3}$$

where ν_{t+1} is an error term that is orthogonal to all variables dated $t-j$ ($j \geq 0$). Since $\rho > 0$ and $c > 0$, we must have $\alpha < 0$ and $\beta > 0$. Regression analysis (see Shiller et al. (1983); Mankiw (1986); Campbell and Shiller (1991)) has always led to the conclusion that the actual change in the long term interest rate tends to be opposite to the theoretical prediction, i.e. $\beta < 0$. But although Mankiw (1986) finds that β is negative for all countries in his data set, it is almost never significantly negative.

The purpose of this note is to show that in empirically relevant cases the population R^2 of Eq. (3) will be extremely low. Consequently the estimates of β will be highly inaccurate. Using a small Monte Carlo study it appears that there is a substantial small sample bias. Moreover, the bias is related to the time series properties of the process followed by the short rate. In order to analyze the properties of least squares estimates of Eq. (3), we first derive coherent time series representations for the spread and the long term interest rate. This is the subject of Section 2 of the paper.

In Section 3 we discuss the results of the Monte Carlo experiment and conclude that regression tests based on Eq. (3) will not provide convincing evidence against the expectations model. Bekaert et al. (1996) considered the small sample properties of this regression. They report substantial biases in the regression coefficients and find that the asymptotic approximations are not to be trusted even with fairly large time series datasets. Moreover, they find that the small sample bias adjustments strengthen the evidence against the expectations hypothesis. Extending Bekaert et al. (1996) we find that the small sample bias can go either way, depending on the time series process for the short term interest rate.

2. Time series representations of the long rate

Any assumed model for the short term interest rate immediately implies a model for the spread and the long term interest rate by calculating the future expectations in Eq. (1). We will assume that the short term interest rate has the following general data generating process:

$$\Delta r_t = c(L)\epsilon_t = \sum_{i=0}^{\infty} c_i \epsilon_{t-i}, \tag{4}$$

where L is the lag operator, c_i are parameters, and ϵ_t are serially uncorrelated errors with mean zero and variance σ^2. It is assumed that $c(1)$ is bounded. If $c(1) = 0$, the lag polynomial $c(L)$ is divisible by the difference operator, meaning that the short rate has been overdifferenced, and a levels model will be appropriate. Using Eq. (4) it will be convenient first to compute the spread S_t, which follows from Eq. (1) after subtracting r_t from both sides and rearranging,

$$S_t = R_t - r_t = \sum_{i=1}^{\infty} \delta^i E_t[\Delta r_{t+i}]. \tag{5}$$

The optimal forecasts of Δr_{t+i} are obtained from Eq. (4):

$$E_t[\Delta r_{t+i}] = \sum_{j=0}^{\infty} c_j \epsilon_{t-j+i}. \tag{6}$$

Substituting Eq. (6) in Eq. (5) one obtains the implied time series representation of the spread. The result is

$$S_t = \sum_{j=0}^{\infty} \sum_{i=1}^{\infty} \delta^i c_{i+j} \epsilon_{t-j}. \tag{7}$$

From the spread S_t we can derive the process for the change in the long term interest rate form $\Delta R_t = \Delta S_t + \Delta r_t$. Using Eq. (4) and Eq. (7) and recollecting terms gives,

$$\Delta R_t = c(\delta)\epsilon_t + (1-\delta) \sum_{j=1}^{\infty} c_j(\delta)\epsilon_{t-j} = g(L)\epsilon_t, \tag{8}$$

where $c_j(\delta) = \Sigma_{i=0}^{\infty} \delta^i c_{j+i}$. The lag polynomial $c(L)$ of the Δr_t process completely specifies the model for the long term interest rate R_t; $g(L)$ is a function of $c(L)$.

Interesting and empirically relevant dynamics are generated by the univariate ARIMA(1, 1, 1) model analyzed in Campbell and Shiller (1984):

$$\Delta r_t = \theta\, \Delta r_{t-1} + \phi \epsilon_{t-1} + \epsilon_t, \tag{9}$$

The Moving Average representation of this process is

$$\begin{array}{l} c_0 = 1 \\ c_1 = \theta + \phi \\ c_j = \theta c_{j-1}, \qquad j > 1 \end{array}. \tag{10}$$

If $\theta + \phi$ is close to zero then Δr_t will be nearly serially uncorrelated at all lags, so the process can produce the typical sample autocorrelations for the short rate. Using Eq. (7) and Eq. (10) the spread becomes the AR(1) process

$$S_t = \theta S_{t-1} + \frac{\delta(\theta+\phi)}{1-\delta\theta}\, \epsilon_t. \tag{11}$$

With θ close to one the ARIMA(1, 1, 1) is the simplest time series model that broadly matches the observed sample autocorrelation function of time series of the short rate and the spread. The long term interest rate follows from Eq. (8):

$$\Delta R_t = \theta\, \Delta R_{t-1} + \psi \nu_{t-1} + \nu_t, \tag{12}$$

where:

$$\nu_t = \frac{1+\delta\phi}{1-\delta\theta}\, \epsilon_t,$$
$$\psi = \frac{\phi(1-\delta\theta) - \delta(\theta+\phi)}{1+\delta\phi}$$

This is also an ARIMA(1, 1, 1) process. If $\theta + \phi$ is close to zero, and θ close to unity, $\theta + \psi$ is close to zero. The theoretical autocorrelation function is also broadly consistent with observed autocorrelations for ΔR_t. It will be difficult to estimate an ARIMA(1, 1) for Δr_t if θ and ϕ are both close to unity but of opposite sign, as the model will hardly be distinguishable from a random walk or the stationary model $r_t = \theta r_{t-1} + \epsilon_t$. Campbell and Shiller (1984), however, find $\theta = 0.95$ and $\phi = -0.975$ for U.S. one month Treasury bill rates.

3. The predictive power of the spread

The theoretical autocovariance functions of Δr_t, S_t, and ΔR_t can be used to investigate the properties of the typical regression Eq. (3) to test the expectations model. For quarterly data reasonable parameter values are $\delta = 0.98$, $\theta = 0.95$, $\phi = -0.975$. With these parameters the population R^2 of Eq. (3) is as small as

$$R^2 = 1 - \frac{\mathrm{Var}(\nu_t)}{\mathrm{Var}(\Delta R_t)} = 0.0013, \tag{13}$$

so that it can be expected to take $3.84/0.0013 = 2953$ observations (about 740 years!) before this R^2 will be judged significant at the 5% level.

The low expected R^2 already partly explains the many inconclusive results in testing the expectations hypothesis. The problems with tests of $\rho > 0$ will be enhanced in small samples because of the multicollinearity between the constant term and the slowly moving spread. Also note that the regression equation is very unbalanced. The left hand side variable ΔR_{t+1} has autocorrelations of the order 0.03 or lower, whereas the right hand side regressor S_t has a first order autocorrelation θ. With θ close to unity, we are regressing almost white noise on a slowly moving variable. A large bias in ρ is to be expected.

Next we perform a small Monte Carlo experiment to investigate the properties of the OLS estimator of β in model Eq. (3), when interest rates are generated by the ARIMA models Eq. (9) and Eq. (12). Table 1 shows the expected $\hat{\beta}$, its standard error, and asymptotic standard error for selected values of δ, θ, ϕ, and sample size T. For quarterly data the value $\delta = 0.98$ corresponds to an annual discount rate of about 8 percent. In columns 1 and 2 of the table we took $\theta = 0.7$, being the first order autocorrelation of the spread with quarterly data. The bias of β is enormous. If $\theta + \phi < 0$ the bias is positive; for $\theta + \phi > 0$, it is negative. In the latter case we still find $\bar{\beta} < 0$ with 450 observations, which is equivalent to 112.5 years.

Columns 3 and 4 pertain to monthly data with same annual discount rate of 8% and the parameter values of Campbell and Shiller (1984). The most dramatic case is $\theta + \phi > 0$ and $T = 50$. The mean of β is -0.30, and in 22% of cases $\beta / s(\beta)$ is smaller than -2, so that one would unjustly tend to reject the expectations theory too often. For sample sizes $T > 150$, however, the test correctly rejects the null hypothesis about 5% of the time when $\theta + \phi > 0$. Bekaert et al. (1996) always report an upward bias, as in columns 2 and 4. The data generating process of Bekaert et al. (1996) is stationary, so that the persistence $c(1) = 0$. Our positive bias also occurs, when the persistence $c(1) = (1 + \phi)/(1 - \theta)$ is less than one. For example, in column 4 with the upward bias we have $c(1) = 0.5$, while in column 3 the persistence is 1.5. But except for the long run persistence, the time series processes for Δr_t are very similar.

Table 1
Monte Carlo results

		Quarterly data $\delta=0.98,\ \rho=0.0204$		Monthly data $\delta=0.995,\ \rho=0.0050$	
		$\theta=0.7$ $\phi=-0.6$	$\theta=0.7$ $\phi=-0.8$	$\theta=0.95$ $\phi=-0.925$	$\theta=0.95$ $\phi=-0.975$
$T=50$	$\bar{\beta}$	−0.2530	0.1642	−0.3023	0.1214
	$s(\hat{\beta})$	0.4893	0.2624	0.3143	0.1666
	$\mathrm{as}(\hat{\beta})$	0.4256	0.2226	0.1414	0.0530
	p	0.06	0.01	0.22	0.04
$T=150$	$\bar{\beta}$	−0.0710	0.0683	−0.0927	0.0435
	$s(\hat{\beta})$	0.2697	0.1424	0.1283	0.0687
	$\mathrm{as}(\hat{\beta})$	0.2451	0.1285	0.0816	0.0306
	p	0.05	0.01	0.12	0.04
$T=450$	$\bar{\beta}$	−0.0050	0.0338	−0.0234	0.0152
	$s(\hat{\beta})$	0.1457	0.0750	0.0568	0.0257
	$\mathrm{as}(\hat{\beta})$	0.1415	0.0742	0.0471	0.0177
	p	0.03	0.01	0.06	0.03

Notes: Monte Carlo results are based on 1000 replications. T is sample size, $\bar{\beta}$ is the average of $\hat{\beta}_i$, ($i=1,\ldots,1000$), with $\hat{\beta}_i$ the OLS estimate of β, $s(\hat{\beta})$ is the standard deviation of $\hat{\beta}_i$, and $\mathrm{as}(\beta)$ is the asymptotic standard error of $\hat{\beta}$, computed as

$$\mathrm{as}(\hat{\beta})=\left(\frac{1-\theta^2}{T}\right)^{1/2}\frac{1+\delta\phi}{\delta|\delta+\phi|}.$$

Finally, p is the rejection frequency of the null hypothesis $\beta>0$ based on $\hat{\beta}/s(\hat{\beta})<-1.96$, where $s(\hat{\beta})$ is the OLS standard error of $\hat{\beta}$.

4. Conclusions

Empirical evidence against the expectations model that relies on the forecasting regression of the change in the long term interest rate is not very convincing, if the short term interest rate follows a dynamic process that is close to a random walk. First, the population R^2 of this regression is extremely low. Second, the OLS slope coefficient can be badly biased, either upward or downward.

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